Image Data Transformation: k-Space

An Offprint from

Peter A. Rinck

Magnetic Resonance in Medicine

A Critical Introduction

The Basic Textbook
of the European Magnetic Resonance Forum

13th edition • 2022
335 figures, 36 tables

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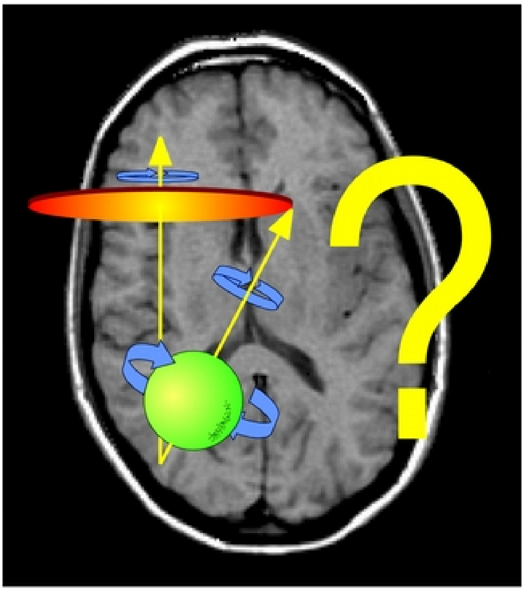
Foreword



"Why, sometimes I've believed as many as six impossible things before breakfast."
The White Queen in Lewis Caroll's 'Alice Through the Looking Glass'.

We like books – printed on paper, if possible with a beautiful hardcover binding. Thus, putting this standard textbook on the internet some years ago was a challenge. Now we return with a printed version of the magnetic resonance textbook. The reasons I have described elsewhere.[1]

Celebrating the 50th anniversary of MR imaging in 2021 was a good occasion to publish a new edition. The textbook-child has grown up, become an adult or, in our case – a rather successful standard textbook. The reviews and public reaction to the book were extremely positive.

The first version of this primer – a little booklet – was written at Paul C. Lauterbur's laboratories in the early 1980s. Lauterbur was the father of MR imaging and received the Nobel Prize twenty years later. The text was intended to be used as the Basic Textbook for EMRF, the European Magnetic Resonance Forum. After Lauterbur saw the first edition, he commented: "It looks like a fine book, especially for residents, nurses, and technicians."

Initially we thought this statement was not very encouraging, but in hindsight this was exactly what we had intended to write. We worked on it for another twenty years – and finally Lauterbur found the last edition he read before his death "gratifying". How-

1 Rinck PA. An expensive dilemma: Tablets versus textbooks. Rinckside 2015; 26,7: 17-19.

ever, the target audience today includes scientists and university professors. They should be able to acquire a basic knowledge which enables them to pursue studies of their own and to cope with some of the most common problems, among them tissue relaxation, image contrast and artifacts or questions concerning possible hazards to patients – and to become aware of how to perform reliable research, and to ask and be critical.

The main author and the contributors have not attempted to cover the field completely nor to be exhaustive in the topics discussed, as the field of magnetic resonance still is in a permanent stage of development and therefore changing year by year. Clinical MR machines and even equipment sold for scientific purposes have been increasingly altered into push-button black boxes with pre-fab, given and unchangeable protocols. We are not interested in certain gadgets or "apps" of commercial machines, and won't mention or describe them. We try to explain the fundamentals any user should know and understand.

As with everything in life, MR imaging does not only require knowledge of facts but also of background information and of the historical development of the field for critical decision making. Therefore we have interspersed some subjective, critical, and opinion-oriented sections – interludes – intended to offset the technical nature of the teaching sections and provide some insights into more practical questions faced by MR users.

Most of them were taken from *Rinckside* (www.rinckside.org), a collection of columns published since 1990.

Many of the recent developments concerning MR equipment and its medical and biological applications have turned away from magnetic resonance itself to novel engineering and software approaches in image processing including artificial intelligence. Techniques, ideas and algorithms were imported from fields outside medicine and adopted by software engineers with little or no background in MR and medicine nor insight into medical needs. We mention some of the prime approaches without going into details of signal or image processing – they are of no importance for the understanding of fundamental facts of magnetic resonance imaging.

There has been a long list of contributors to this and earlier versions (see page 418). Their support, ideas, dedication, and feedback have added much to the quality of this work. This book was peer-reviewed by a number of competent reviewers in different fields whom I thank for their efforts.

If you want to learn something about magnetic resonance imaging or its applications choose your topic of interest. If you want to learn it from scratch start with Chapter 1; and if you want to air your brain, read the interludes that are scattered in between.

If you find any mistakes in this book, rest assured that they were left intentionally so as not to provoke the gods with something which is perfect. Still, we would be happy about your feedback. We hope that this textbook will be useful for you and that you will enjoy it. If you have comments or suggestions, please write to us.

Peter A. Rinck, July 2021

Contents

Chapter Seven

Image Data Transformation: k-Space

Figure 07-01:
There is something wrong here: we do not talk about CAT-scans in CAT-space – or do we?
The picture on the left was taken during daytime, the picture on the right at night. Look at the cat's eyes: the pupils are small when there is a lot of light, but then they are wide with little light.
The central part of the retina displays extraordinary visual discrimination, thanks to the tiny size of the closely packed, light sensitive cones located there. This area with maximum resolution covers only 1° of the eye's field-of-view. At night, the periphery of the retina is used; it has an incredible sensitivity to light but a very poor ability to distinguish details.

Introduction

Its flexibility distinguishes MR imaging from all other medical imaging modalities. The ultimate reason for this is the unique handling of MR raw data in an abstract data collection matrix called *k-space*, where the data stay to be deciphered.

This space consists of the raw data that have been collected during image acquisition but have not yet been converted into the final anatomical image.

The motto in the foreword to this book fits very nicely with this chapter. The easiest way to deal with k-space is seeing and believing; this, however, is not very helpful when one wants to understand how some imaging techniques function and what their pitfalls are (Figure 07-01).

Of course, k-space behaves differently from cat eyes, but there are some similarities as will be explained in the text.

First and foremost, a k-space is a mental concept. There is no hardware in an MR machine corresponding to it. It is a platform to collect, store, and process complex data. These data represent thousands of sine and cosine waves which build the MR image.

The term *k-space* is mathematical. The letter 'k' is used by mathematicians and physicists to describe spatial frequency, for instance, in the propagation of sound, light, or, in general, electromagnetic waves.

The Optical Equivalent

One way of understanding the concepts and mechanisms of k-space is looking at a different physical property which, perhaps, is simpler to imagine: the collection and processing of light by a lens, as Mezrich explains in his introduction to k-space.[101]

The processing of the incoming light to an image by the lens determines to a great extent its resolution, size, and contrast. The light passing through the lens is bent slightly in the center, increasingly towards the edges. In a perfect lens, the light will meet in one point, the focus, and then create an inverted image (Figure 07-02).

The processing of the light data by a lens is more complicated than generally thought: there is no point-to-point correspondence between points within the lens – or within a center plane in the middle of the lens – and the final image created by the lens. All points within the lens process data from all points of the original object. However, for our purposes we could imagine such a center plane as the location where processing takes place (Figure 07-03).

Visible light actually consists of different frequencies. As we have seen in Chapter 2, a prism can make a frequency analysis. A lens is more sophisticated. We can consider it as a special filter which, depending on its characteristics, lets some or all of these frequencies pass. It accepts signals, analyses them, processes them, and creates an image; basically, it performs a Fourier transform. We have assumed that the Fourier transform is accomplished in a fictitious central plane of the lens. In front of the lens, we can set instruments performing optical functions, for instance an iris, or we can change the size of a lens (Figure 07-04).

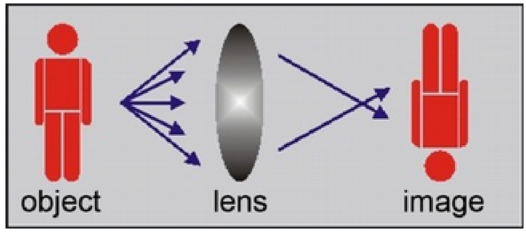


Figure 07-02:
Image processing by a lens.

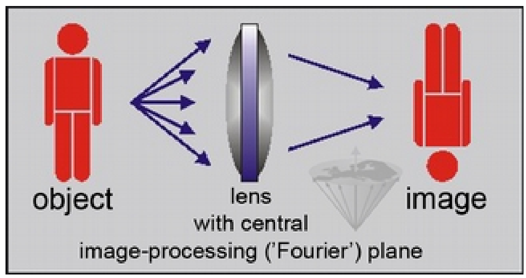


Figure 07-03:
Image processing by a lens with a fictitious image-processing plane.

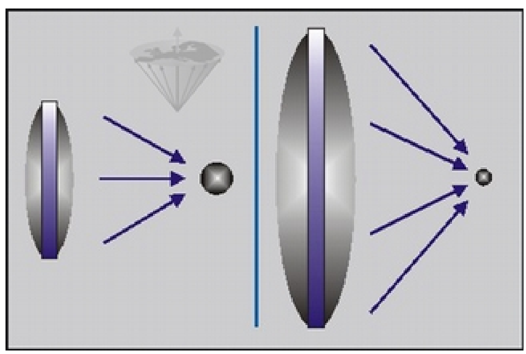


Figure 07-04:
Increasing the size of a lens with the same focus improves image resolution because the individual image points are smaller – the same holds for k-space: larger k-space with the same field-of-view means better spatial resolution of the image.

101 Mezrich R. A perspective on k-space. Radiology 1995; 195: 297-315 [review].

Changing the size of the lens or an iris influences the size of our processing plane. The steeper the angle the light makes within the lens, the sharper the focus will be; the larger the lens, the better image resolution will be. The sharpness of the final image is determined by the outer parts of our 'Fourier' plane. Points in the outer regions of the plane contribute more to image resolution than points close to the center because they allow higher spatial frequencies to pass through.

Lower spatial frequencies are closer to the center. Their main responsibility is the distribution of brightness and darkness. This means that they are responsible for image contrast.

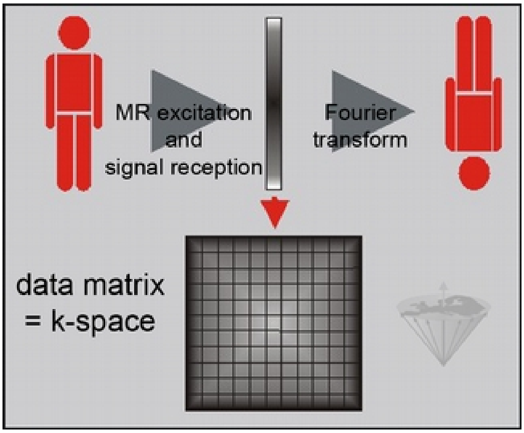


Figure 07-05:
From lens to MR imaging. If you compare this figure with Figure 06-20, you would position k-space before the second Fourier transform.

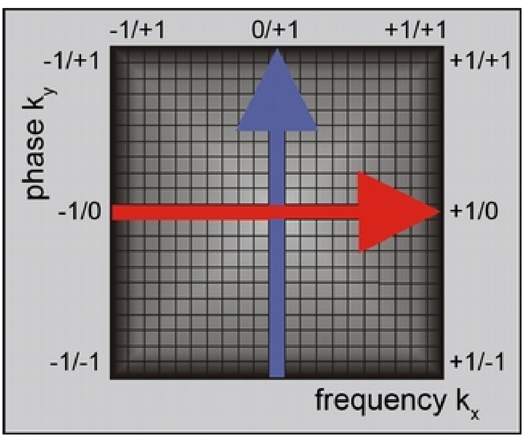


Figure 07-06:
The k-space raw data matrix consists of an area to be filled with the information needed to form an MR image.

MR Imaging and k-Space

What we have said about optical lenses holds, in a similar way, for k-space in MR imaging (Figure 07-05).

As the lens, k-space collects image raw data for Fourier transform. One of the main differences is the shape: lenses are round, k-space is rectangular. In k-space, the iris of the camera is replaced by gradient strength, in one direction for frequency-, in the other direction for phase-encoding (Figure 07-06).

The coordinates of k-space are called *spatial frequencies* (measured in cycles per millimeter). They are filled depending on gradient strength of the frequency-encoding gradient (*readout gradient*: red arrow; x-direction) and phase-encoding gradient (*preparation gradient*: blue arrow; y-direction), moving from low gradient strength (-1) to zero gradient strength in the center (0) and high gradient strength (+1).

In MR imaging, k is divided into three dimensions (k_x, k_y, and k_z) which define a domain or a space. Only two of them are commonly included, k_x and k_y. The third, k_z, is the slice-selecting gradient which is mostly disregarded in k-space.

Figure 07-07 (right):
k-Space with spatial frequency filtering.

Figures a1 and a2: Regular k-space with image reconstruction.

Figures b1 and b2: Same k-space as in (a) with filtering of the high frequencies; the reconstructed image has lost sharpness, it looks blurred; however, image contrast has hardly been affected.

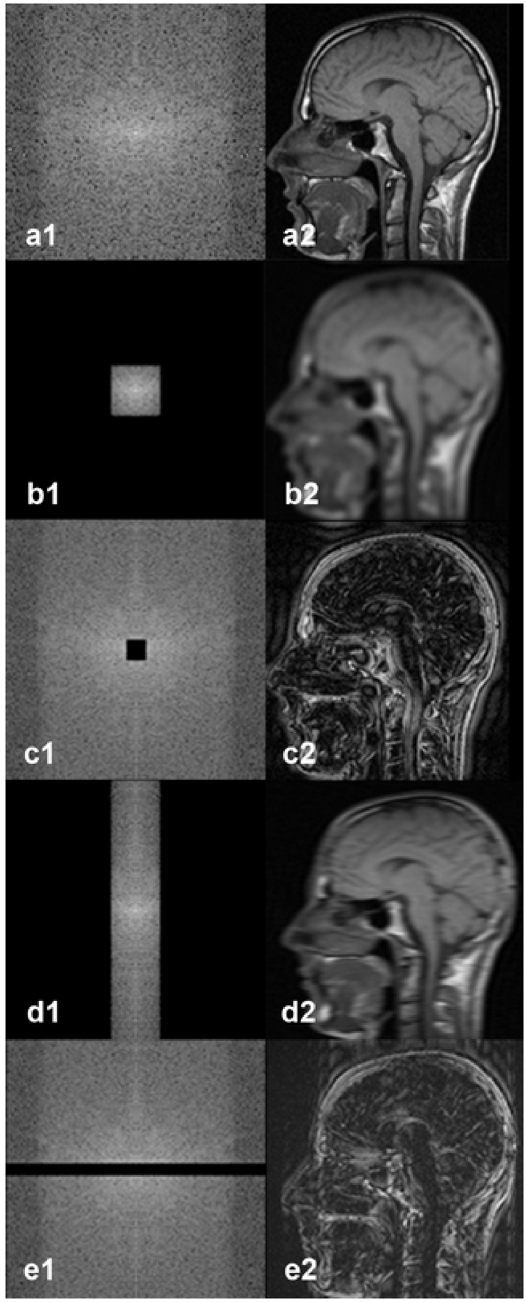
Figures c1 and c2: Spatial frequency filtering of the low frequencies; the reconstructed image has lost image contrast, but image details have hardly been affected.

Figures d1 and d2: Low pass filtering in the readout direction, …

Figures e1 and e2: ... high pass filtering in the preparation direction.

The signal amplitude (or magnitude) corresponds with the absolute brightness on the image because k-space is the representation of the amplitudes of the sampled echoes. Therefore, the highest intensity is in the center.

Simulation software: MR Image Expert®



The points at the center of this raw data matrix represent small gradients; increasing the offset from the center corresponds to increasing gradient strength. [102, 103] Again, in an MR image the low spatial frequencies determine the gross signal levels (and hence contrast), while the higher spatial frequencies principally determine the edge definition (sharpness), as shown in Figure 07-07. The definition of small objects is an integral part of the contrast and requires highspatial frequencies; thus, in this situa-

102 Ljunggren S. A simple graphical representation of Fourier-based imaging methods. J Magn Reson 1983; 54: 338-343.

103 Twieg DB. The k-trajectory formulation of the NMR imaging process with applications in analysis and synthesis of imaging methods. Med Phys 1983; 10: 610-621.

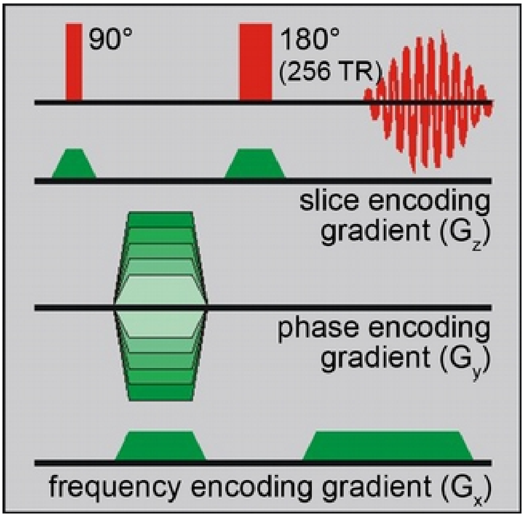


Figure 07-08:
Graphic depiction of a spin-echo pulse sequence.

tion the high spatial frequencies also contribute to contrast.

The maximum signal intensity is recorded close to the center of k-space since the net read and phase gradients applied for these points are relatively small, resulting in less dephasing.

Filling k-Space with Data and Image Reconstruction

In most MR imaging sequences applied in clinical routine today, raw data are placed in a rectangular k-space grid.[104]

In a standard spin-echo sequence, each 90° pulse creates a new line (Figure 07-08). The length of the line is determined by the strength of the frequency-encoding gradient and the sampling time, its position by the strength of the phase-encoding gradient.

The position of the line is determined as follows:

After the initial 90° excitation pulse, the spins evolve in the direction given by the phase-encoding gradient Gy and the frequency-encoding gradient Gx (yellow arrow in Figure 07-09a, overleaf).

They are then turned around by the 180° pulse (red/magenta arrow). Then the frequency encoding gradient is switched on again and sampling starts. This is repeated for different amplitudes of the phase-encoding gradient until k-space is filled (Figure 07-09b, overleaf).

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104 Edelstein WA, Hutchison JMS, Johnson G and Redpath TW. Spin warp NMR imaging. Phys Med Biol 1980; 25: 751-756.

The time needed for such an imaging experiment is the number of phase-encoding steps (NGy) multiplied by the repetition time (TR) and the number of excitations (NEX):

$$t = NGy \times TR \times NEX$$

Now we have filled the data matrix with each row containing information from one echo. Each data point is then Fourier-transformed in the x-direction, which leads to a new data matrix where every point in each column contains information stemming from a certain frequency; the phase information differs point-by-point per row.

The second Fourier transform is performed in the y-direction to extract phase information.

This again leads to a new data matrix containing combined phase and frequency information. The output is a matrix showing a *modulus* or *magnitude* image which corresponds to the bulk of MR signals from each point. Phase correction might be necessary to correct for phase jumps between 0° and 360°.

Among the main parameters influenced by k-space are the speed of acquisition, spatial resolution, field-of-view, contrast, and artifacts. Details can be found in some dedicated papers.[105, 106, 107, 108]

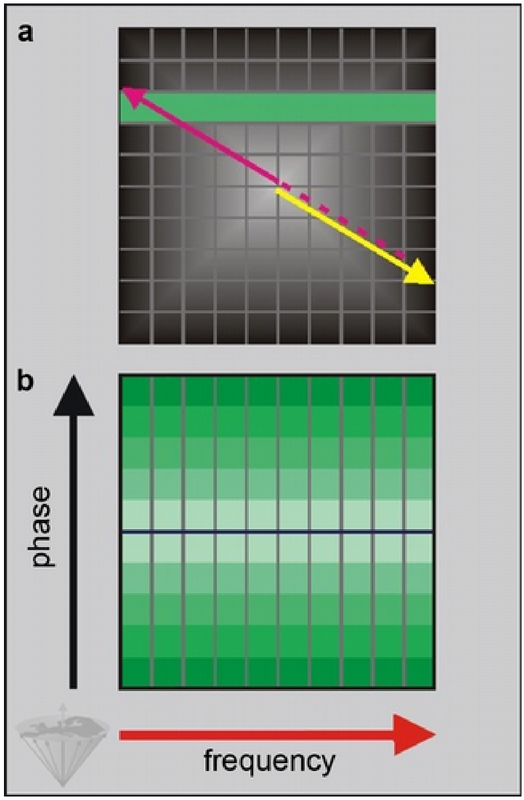

Figure 07-09:
Mapping of k-space in a spin-echo pulse sequence. **(a)** Positioning of a single line. **(b)** Filling of the entire k-space. Phase direction: blue arrow, frequency direction: red arrow.
In conventional pulse sequences, such as the spin-echo sequence, one line of k- space is filled per repetition time (TR) cycle (commonly there are 256 cycles per imaging experiment, not only ten as in this figure).

105 Hennig J. K-space sampling strategies. Eur Radiol 1999; 9: 1020-1031 [review].
106 Mezrich R. A perspective on k-space. Radiology 1995; 195: 297-315 [review].
107 Pelc NJ, Glover GH. A stroll through k-space. In: Medical Physics Monograph no. 21: The Physics of MRI. American Institute of Physiscs 1993; 21: 771 [review].
108 Peters TM. An introduction to k-space. In: Medical Physics Monograph no. 21: The physics of MRI. American Institute of Physiscs 1993; 21: 754 [review].

The Author

Peter A. Rinck is a University Professor of Radiology and Magnetic Resonance (*emeritus*) and has a Doctorate in History of Medicine.

After a classical school education he attended medical school in Berlin (Free University of Berlin) and served his internship and residency in radiology, nuclear medicine and radiation therapy at Charlottenburg University Hospital in Berlin.

Afterwards, until 1983, he was involved in the very early development of magnetic resonance imaging as Senior Research Associate at the State University of New York at Stony Brook where he worked in Paul C. Lauterbur's research group (Nobel Prize in Medicine 2003). The first version of this textbook was written at this time.

Subsequently Rinck worked as physician-in-charge of one of the first two German government sponsored MR machines in Wiesbaden, Germany.

Between 1987 and 1994 he was head of Europe's biggest clinical and research MR facility – at that time – at the University of Trondheim, Norway. Between 1986 and 2012 he was also Adjunct Professor at the School of Medicine and Pharmacy of the University of Mons-Hainaut in Belgium.

Since 1982 Rinck is Chairman of the European Magnetic Resonance Forum, EMRF, and since 2008 President of the Council of The Round Table Foundation, TRTF.

He is also Chairman of the Selection Committees of the the Pro Academia Prize and of the European Magnetic Resonance Award.

Visiting Professorships: The Neurological Institute of Colombia. Bogotá, Colombia (1986); Charité University Hospital, Medical Faculty of Humboldt University, Berlin, Germany (1991-1992); et al.

President of the European Society for Magnetic Resonance in Medicine and Biology, 1985-1987; president of the annual meetings 1989, 2002. Scientific consultant and expert adviser to international organizations and foundations (among them WHO, European Commission, UNIDO, the Nobel Committee). Honorary, founding, or ordinary member of numerous professional and learned societies.

Among others, awards and prizes from the Alexander von Humboldt Foundation, Max Kade Foundation, NATO, European Commission, Fonds National de la Recherche Scientifique de Belgique, the Research Council of Norway, and German Research Society (DFG).

Author and/or editor of several books – not only scientific or medical – an e-learning website, numerous papers in refereed journals and communications to international scientific meetings; and since 1990 *Rinckside* (learned columns).

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